Introducing Connection Minimal Abduction for $\mathcal{EL}$ Ontologies *

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Abstract. We introduce a new, explanation-oriented, minimality notion for abduction in the description logic $\mathcal{EL}$. It rejects solutions where a random concept inclusion, disconnected from the problem at hand, is used.

Keywords: Abduction · Description Logic $\mathcal{EL}$· Minimality Criterion.

1 Introduction

Ontologies are used in many areas to represent and reason about terminological knowledge. Usually, they consist of a set of axioms formulated in a description logic (DL), giving definitions of concepts, or stating relations between them. Especially in the bio-medical domain, we often find ontologies that contain a lot of those axioms, for instance, SNOMED CT[4] containing over 350,000 axioms, or the Gene Ontology GO[5] defining over 50,000 concepts. A central reasoning task for ontologies is to determine whether one concept is subsumed by another, a question that nowadays can be answered rather efficiently using highly optimized description logic reasoners [23]. If the answer to this question is unexpected or hints at an error, a natural interest is in an explanation for that answer—especially if the ontology is complex. Such an explanation can help in understanding the internal mechanisms of an ontology, as well as facilitate debugging. Explaining positive entailments—e.g., explaining why a concept subsumption holds—is well-researched in the DL lecture, and typical approaches use justifications [5, 17, 26] or proofs [1, 2], functionalities that are integrated into standard ontology editors [18, 19]. The problem of explaining negative entailments—e.g. explaining why a subsumption does not hold—is less investigated, and there is no standard tool support. Classical approaches involve providing a counter-example [6], or using *abduction*.

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[4] https://www.snomed.org/

[5] http://geneontology.org/

In abduction, a missing entailment $\mathcal{T} \not\models \alpha$ is explained by providing a "missing piece", the *hypothesis*, that, when added to the ontology, would entail the answer. It can thus be used to explain why the entailment does not hold, and even provide for a possible fix in case the entailment should hold. In the DL context, depending on the shape of the (missing) observation to be explained, one distinguishes between concept abduction [7], ABox abduction [8–11, 14, 16, 20, 21, 24, 25], TBox abduction [12, 27] or knowledge base abduction [15, 22]. We are here focussing on TBox abduction, where we assume that ontology, observation and hypothesis consist only of TBox axioms.

Commonly, to avoid trivial answers, the user provides syntactic restrictions on hypotheses, such as a set of abducible axioms to pick from [9, 24], a set of abducible predicates [21, 22], or even patterns on the shape of the answer [13]. But even with those restrictions in place, there may be many possible solutions to an abduction problem, and the question is how to find the ones with the best explanatory potential. It is here common to either provide a representation that covers all explanations at the same time [22], or to apply minimality criteria such as subset minimality, size minimality, or semantic minimality [8] to prune the set of solutions. We argue that using those more superficial minimality criteria may still lead to hypotheses that are not that helpful, as they ignore the deeper logical structure of the hypotheses and their connection to the observation. To address this issue, we introduce a new minimality criterion called *connection minimality*, which we define for the lightweight description logic $\mathcal{EL}$.

2 Preliminaries

We recall the description logic $\mathcal{EL}$ and provide a formal definition of the abduction problem as used in this abstract.

Let $\mathsf{N_C}$ and $\mathsf{N_R}$ be pair-wise disjoint, countably infinite sets of respectively *atomic concepts* and *roles*. Generally, we use letters A, B, E, F... for atomic concepts, and r for roles, possibly annotated. Letters C, D, possibly annotated, denote $\mathcal{EL}$ *concepts*, built according to the syntax rule $C ::= \top \mid A \mid C \sqcap C \mid \exists r.C$. We assume that conjunctions are treated as sets, that is, we ignore nested conjunctions, and assume that the order of conjuncts is not important. For a set N of concepts, we denote by $\bigsqcap N$ the conjunction over all elements in N. An $\mathcal{EL}$ *TBox* $\mathcal{T}$ is a finite set of *concept inclusions* of the form $C \sqsubseteq D$. We use $C \equiv D$ as abbreviation for $C \sqsubseteq D$, $D \sqsubseteq C$. The semantics of $\mathcal{EL}$ is defined as usual (see e.g. [3]). Specifically, for a TBox $\mathcal{T}$, we use $\mathcal{T} \models C \sqsubseteq D$ to denote that $C \sqsubseteq D$ holds in all models of $\mathcal{T}$, in which case we also say $C \sqsubseteq D$ is *entailed* by $\mathcal{T}$.

Definition 1. *Given a TBox $\mathcal{T}$, a set of atomic concepts $\Sigma \subseteq \mathsf{N_C}$ and a concept inclusion $C_1 \sqsubseteq C_2$, denoted as the* observation*, where C_1 and C_2 are atomic concepts and $\mathcal{T} \not\models C_1 \sqsubseteq C_2$, the corresponding* TBox abduction problem*, denoted as the tuple $\langle \mathcal{T}, \Sigma, C_1 \sqsubseteq C_2 \rangle$, is to find a TBox*

$$\mathcal{H} \subseteq \{A_{i_1} \sqcap \cdots \sqcap A_{i_n} \sqsubseteq B_{i_1} \sqcap \cdots \sqcap B_{i_m} | \{A_{i_1}, \ldots, A_{i_n}, B_{i_1}, \ldots, B_{i_m}\} \subseteq \Sigma\}$$

such that $\mathcal{T} \cup \mathcal{H} \models C_1 \sqsubseteq C_2$ *and* $\mathcal{T} \cap \mathcal{H} = \emptyset$. *The solution* $\mathcal{H}$ *to the abduction problem is denoted as a* hypothesis.

Note that since $\mathcal{EL}$ TBoxes are always consistent, the consistency condition usually required on $\mathcal{T} \cup \mathcal{H}$ is not needed here.

3 Limitation of Existing Criteria

We illustrate the problem that arises with the existing criteria in the following example. Given a TBox

$$\mathcal{T} = \{ \exists\mathsf{employment}.\mathsf{ResearchPosition} \sqcap \exists\mathsf{qualification}.\mathsf{Diploma} \sqsubseteq \mathsf{Researcher}, \exists\mathsf{writes}.\mathsf{ResearchPaper} \sqsubseteq \mathsf{Researcher}, \mathsf{Doctor} \sqsubseteq \exists\mathsf{qualification}.\mathsf{PhD}, \mathsf{Professor} \equiv \mathsf{Doctor} \sqcap \exists\mathsf{employment}.\mathsf{Chair}, \mathsf{FundsProvider} \sqsubseteq \exists\mathsf{writes}.\mathsf{GrantApplication}\}$$

and an observation $\textsc{Obs} = \mathsf{Professor} \sqsubseteq \mathsf{Researcher}$, the TBoxes

$$\mathcal{H}_1 = \{ \mathsf{Chair} \sqsubseteq \mathsf{ResearchPosition}, \mathsf{PhD} \sqsubseteq \mathsf{Diploma}\} \text{ and}$$
$$\mathcal{H}_2 = \{ \mathsf{Professor} \sqsubseteq \mathsf{FundsProvider}, \mathsf{GrantApplication} \sqsubseteq \mathsf{ResearchPaper}\}$$

are solutions of the TBox abduction problem $\langle \mathcal{T}, \mathsf{N_C}, \textsc{Obs} \rangle$. Note that both solutions are subset minimal, have the same size, and incomparable wrt. the entailment relation, so that traditional minimality criteria cannot distinguish them. However, intuitively, the second hypothesis feels more arbitrary than the first. Specifically, looking at $\mathcal{H}_1$, Chair and ResearchPosition occur in $\mathcal{T}$ in concept inclusions where the concepts in Obs also occur, and both PhD and Diploma are similarly related to Obs but via the role qualification. In contrast, $\mathcal{H}_2$ involves the concepts FundsProvider and GrantApplication that are not related to Obs in any way in $\mathcal{T}$.

In fact, any random concept inclusion $A \sqsubseteq \exists\mathsf{writes}.B$ in $\mathcal{T}$ would lead to a hypothesis similar to $\mathcal{H}_2$ where A replaces FundsProvider and B replaces GrantApplication. Hence, the explanatory power of such hypotheses is very low. We need a minimality criterion that can discard hypotheses like $\mathcal{H}_2$ if we want to keep only solutions with high explanatory potential.

4 Connection Minimal Abduction

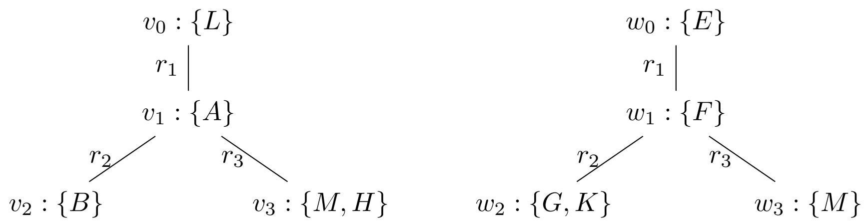
We introduce *connection minimality*, a minimality notion that rejects hypotheses connecting concepts in the observation without care for the existing entailment structure of the TBox. To represent this structure, we rely on the notion of an $\mathcal{EL}$-description tree, originally from Baader et al. [4], which basically correspond to syntax trees of $\mathcal{EL}$ concepts, but ignoring the order of conjunction.

Definition 2. *An* $\mathcal{EL}$-description tree *is a tree of the form* $T = (V, E, v_0, l)$ *with root* $v_0 \in V$ *where*

– *the nodes* $v \in V$ *are labeled with* $l(v) \subseteq \mathsf{N_C}$*, and*
– *the (directed) edges* $vrw \in E$ *are such that* $v, w \in V$ *and are labeled with* $r \in \mathsf{N_R}$.

Given such an $\mathcal{EL}$*-description tree* T*, the* corresponding concept C_T *is defined recursively using* $C_T = C_{T,v_0}$ *and* $C_{T,v} = \bigsqcap l(v) \sqcap \bigsqcap\{\exists r.C_{T,w} \mid vrw \in E\}$.

Example 3. Let $D_1 = L \sqcap \exists r_1.(A \sqcap \exists r_2.B \sqcap \exists r_3.(M \sqcap H))$ and $D_2 = E \sqcap \exists r_1.(F \sqcap \exists r_2.(G \sqcap K) \sqcap \exists r_3.M)$. The respective $\mathcal{EL}$-description trees representing these concepts are



To characterize connection minimal abduction, we map these trees to one another using the following "semantic" notion of homomorphisms.

Definition 4. *Let* $T_1 = (V_1, E_1, v_0, l_1)$ *and* $T_2 = (V_2, E_2, w_0, l_2)$ *be* $\mathcal{EL}$*-description trees. A mapping* $\phi : V_2 \to V_1$ *is a* concept homomorphism *from* T_2 *to* T_1 *w.r.t.* $\mathcal{T}$ *if and only if the following conditions are satisfied:*

1. $\phi(w_0) = v_0$
2. $\phi(v)r\phi(w) \in E_1$ *for all* $vrw \in E_2$
3. *for every* $v \in V_1$ *and* $w \in V_2$ *with* $v = \phi(w)$*,* $\mathcal{T} \models \bigsqcap l_1(v) \sqsubseteq \bigsqcap l_2(w)$

Concept homomorphism w.r.t. a given TBox $\mathcal{T}$ captures entailment w.r.t. $\mathcal{T}$: If ϕ is a concept homomorphism from T_2 to T_1 w.r.t. $\mathcal{T}$ then $\mathcal{T} \models C_{T_1} \sqsubseteq C_{T_2}$. This holds trivially from Point 1 and 3 of Definition 4.

Example 5. Consider again the trees T_1 and T_2 and respective concepts D_1 $(= L \sqcap \exists r_1.(A \sqcap \exists r_2.B \sqcap \exists r_3.(M \sqcap H)))$ and D_2 $(= E \sqcap \exists r_1.(F \sqcap \exists r_2.(G \sqcap K) \sqcap \exists r_3.M))$ from Example 3. Given the abduction problem $\langle \mathcal{T}, \mathsf{N_C}, C_1 \sqsubseteq C_2 \rangle$, where

$$\mathcal{T} = \{\ C_1 \sqsubseteq D_1,\ D_2 \sqsubseteq C_2,\ A \sqsubseteq F,\ B \sqsubseteq G\},$$

and the TBox $\mathcal{H} = \{L \sqsubseteq E, B \sqsubseteq G \sqcap K\}$, the function ϕ such that $\phi(w_i) = v_i$ is a concept homomorphism from T_2 to T_1 w.r.t. $\mathcal{T} \cup \mathcal{H}$. Thus $\mathcal{T} \cup \mathcal{H} \models D_1 \sqsubseteq D_2$. In addition $\mathcal{T} \models C_1 \sqsubseteq D_1$ and $\mathcal{T} \models D_2 \sqsubseteq C_2$. Thus, the TBox $\mathcal{H}$ is a hypothesis.

We use concept homomorphism to characterize entailment w.r.t. a given TBox—as in the example above—and to characterize hypotheses. To obtain our minimality notion, we additionally require solutions to avoid unnecessary conjuncts, which we do using the following two definitions.

Definition 6. *Let* C *and* D *be concepts. Then* $C \preceq_{\sqcap} D$ *if and only if:*

– *$D = C$, or*
– *there exists D' and D'' such that $D = D' \sqcap D''$, and $C \preceq_\sqcap D'$, or*
– *there exists r, C' and D' such that $C = \exists r.C'$, $D = \exists r.D'$ and $C' \preceq_\sqcap D'$.*

Intuitively, $C \preceq_\sqcap D$ if we can remove any number of conjunct subexpressions from D to get C, e.g., $\exists r'.B \preceq_\sqcap \exists r.A \sqcap \exists r'.(B \sqcap B')$.

Definition 7. *Let C_1 and C_2 be concepts. A concept D* connects *C_1 to C_2 in $\mathcal{T}$ if and only if $\mathcal{T} \models C_1 \sqsubseteq D$ and $\mathcal{T} \models D \sqsubseteq C_2$. The* connecting *concept D is* $\preceq_\sqcap$-minimal *if all concepts D' that connect C_1 to C_2 in $\mathcal{T}$ and verifying $D' \preceq_\sqcap D$ are such that $D \preceq_\sqcap D'$.*

For example, assume that both $D = A \sqcap \exists r.B$ and $D' = A \sqcap \exists r.(B \sqcap B')$ connect C_1 and C_2. In that case, we prefer D because $D \preceq_\sqcap D'$ but $D' \not\preceq_\sqcap D$. Note also that if either C_1 or C_2 does not connect C_1 to C_2 in $\mathcal{T}$, then no concept connects C_1 to C_2 in $\mathcal{T}$.

In Example 5, both D_1 and D_2 are connecting concepts in $\mathcal{T} \cup \mathcal{H}$, but $D_1 (= L \sqcap \exists r_1.(A \sqcap \exists r_2.B \sqcap \exists r_3.(M \sqcap H)))$ is not $\preceq_\sqcap$-minimal, because $D'_1 = L \sqcap \exists r_1.(A \sqcap \exists r_2.B \sqcap \exists r_3.M)$ is such that $D'_1 \preceq_\sqcap D_1$ and it also connects C_1 to C_2 in $\mathcal{T} \cup \mathcal{H}$. In fact, D'_1 as well as D_2 are $\preceq_\sqcap$-minimal.

Intuitively, our characterization of minimal hypotheses for abduction problems $\langle \mathcal{T}, \Sigma, C_1 \sqsubseteq C_2 \rangle$ works as follows. As any hypothesis, $\mathcal{H}$ turns some subsumer C_{T_1} of C_1 and some subsumee C_{T_2} of C_2 into connecting concepts from C_1 to C_2 (for some $\mathcal{EL}$-description trees T_1 and T_2). We require that C_{T_1} and C_{T_2} are $\preceq_\sqcap$-minimal, and thus contain no unnecessary conjuncts. Furthermore, we require that the connection between C_{T_1} and C_{T_2} is characterized by a concept homomorphism, and that $\mathcal{H}$ contains only GCIs that are required for this connection to hold. This last condition reflects our idea of *minimal connectedness* motivated in Section 3.

Definition 8 (Connection Minimal Abduction). *Given an abduction problem $\langle \mathcal{T}, \Sigma, C_1 \sqsubseteq C_2 \rangle$, the hypothesis $\mathcal{H}$ is* connection minimal *if and only if there exist $\mathcal{EL}$-description trees $T_1 = (V_1, E_1, v_1, l_1)$ and $T_2 = (V_2, E_2, v_2, l_2)$ s.t.*

1. *$\mathcal{T} \models C_1 \sqsubseteq C_{T_1}, C_{T_2} \sqsubseteq C_2$,*
2. *C_{T_1} and C_{T_2} are $\preceq_\sqcap$-minimal connecting concepts from C_1 to C_2 in $\mathcal{T} \cup \mathcal{H}$,*
3. *there is a concept homomorphism ϕ from T_1 to T_2 w.r.t. $\mathcal{T} \cup \mathcal{H}$, s.t. $\mathcal{H} = \{\bigsqcap l_1(v) \sqsubseteq \bigsqcap l_2(w) \mid w \in V_2,\ v = \phi(w),\ \mathcal{T} \not\models \bigsqcap l_1(v) \sqsubseteq \bigsqcap l_2(w)\}$.*

Returning to Example 5, D'_1 and D_2 satisfy all the requirements of connection minimality, thus the hypothesis $\mathcal{H} = \{L \sqsubseteq E, B \sqsubseteq G \sqcap K\}$ is minimal. This definition also rejects $\mathcal{H}_2$ from Section 3 because it is impossible to find T_1 and T_2 verifying points 1 and 2 of Definition 8. However $\mathcal{H}_1$ is accepted. The connecting concepts $D_1 = \exists \mathsf{employment}.\mathsf{Chair} \sqcap \exists \mathsf{qualification}.\mathsf{PhD}$ and $D_2 = \exists \mathsf{employment}.\mathsf{ResearchPosition} \sqcap \exists \mathsf{qualification}.\mathsf{Diploma}$ are the witnesses attesting that $\mathcal{H}_1$ is a connection minimal hypothesis.

This new notion of minimality achieves our aim of discriminating the more arbitrary solutions. It remains to compute it efficiently. We believe this can be achieved via a translation to first-order logic, but a proper investigation remains to be conducted.

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